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Steady State Analysis of a Three Stage Communication Network with DBA and Batch Arrivals

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Abstract— **The scalability of Internet is leading several problems in delivering the packets from the sender to the receiver. To address these problems, advanced methodologies are to be devised for routing. Increasing the performance of the router/switching nodes and effective utilization of the network resource can also reduces the problem and increases the network performance. In this paper, a three stage communication network model with homogeneous batch arrivals and dynamic bandwidth is developed and analyzed. The data packets after getting transmitted from the first node are forwarded to the second buffer connected to the second node and the packets leaving the second node are forwarded to the third node. Dynamic Bandwidth Allocation (DBA) is the strategy that the transmission rate at each node is adjusted depending upon the content of the buffer at every packet transmission. It is assumed that the arrival of packets follow compound Poisson processes and the transmission completions at each node follow Poisson processes. This model is more accurately fit into the realistic situation of the communication network having a predecessor and successor nodes for the middle node. Using the difference – differential equations, the joint probability generating function of the number of packets in each buffer is derived. The performance measures like, the probability of emptiness of the three buffers, the mean content in each buffer, mean delays in buffers, throughput etc. are derived explicitly under steady state conditions.**

Keywords— **Three stage communication networks, Dynamic bandwidth allocation, Batch arrivals, Steady state and Performance analysis.**

I. INTRODUCTION

The explosive growth of communication services such as Internet, wireless communications, mobile communications, satellite communications, computer communications etc., suggest that the today's environment of communication network is changing drastically and entering in to the advanced information age. The change in the communication network is due to technological advancement and customer needs for quality of service (QoS). The implementation of infrastructure technologies which support the basis of communication network such as transmission, switching, routing, controls etc., have been promoted to match the changes and utilizing the resources more optimally and efficiently. Toshiki Tanak, D.E. et al,(1998) have presented the evaluation of communication networks. O'Brien(1954), Jackson(1954,1956) and Edger Reich(1957) have pioneered the tandem queuing models. Welesey and Alan(1972) discussed the computer communications through computer traffic and channel characteristics, optimal fixed message block size, stastical multiplexing and loop systems. A unified model was developed and used to analyze the queuing behavior of the star and loop systems. The performance these systems has analyzed by using the numerical results for selected traffic intensities and message lengths. The publication of Burke (1972), Reynolds(1975) and Daley(1976) are excellent reviews of literature on output process and tandem queuing models. Hsu and Burke (1976) studied infinity capacity buffers in tandem in which the first buffer consists of individual arrivals characterized by a geometric distribution of the time between arrivals. They have shown in steady state the input process of the subsequent buffers in the system is geometric with the same parameter as a input process of the first buffer. Hisashi and Alan(1977) presented a review on queuing network models and discrete time queuing systems. A unified treatment of buffer storage overflow problems was discussed as application of various multiplexing techniques and network configuration. The technological advancement and innovations in the network equipment lead to the design and development of effective communication networks with packet switching. The large volumes of data originated at different sources at different users is to be delivered with high performance rates through the network, thus the design and analysis of load dependent networks and effective utilization of transmission bandwidth on the transmission lines are major issues of the communication systems (Srinivasa Rao K. et al (2006)). The analysis of statistical multiplexing of data/voice transmission through congestion control strategies are important for efficient utilization of network resources, congestion control is achieved usually by applying bit dropping method. In order to reduce the transmission time a portion of the least significant bits are discarded in the bit dropping method when there is congestion in buffering. While maintaining quality of service expected by the end users. Input bit dropping (IBD) and output bit dropping (OBD) are the usual bit dropping methods (Kin K. Leung, (2002)).

In input bit dropping, bits may be dropped when packets are placed in the queue waiting for transmission. In output bit dropping, bits are discarded when a packet is being transmitted over the channel. This implies fluctuation in voice quality due to dynamically vary bit rate during the transmission (Karanam V.R et al (1988)). For efficient transmission some algorithms have been developed with various protocols and allocation strategies for optimum utilization of the bandwidth for an efficient transmission (Emre and Ezhan, 2008; Gundale and Yardi, 2008; Hongwang and Yufan, 2009; Fen Zhou et al. 2009; Stanislav, 2009). These strategies are developed based on flow control and bit dropping techniques. Some work has been initiated in literature regarding utilization of the idle bandwidth by adjusting the transmission rate instantaneously before transmission of a packet.

Dynamic bandwidth is the transmission strategy of adjusting the data transmission rate depending upon the content of the buffer connected to the node. Suresh Varma et al. (2007) have designed and developed a two node communication network with load dependent transmission their consideration of single packet arrivals to the source node is realistic. Generally, in any communication system these messages arrived to the source node are converted into a random number of packets based on the message size and thus the consideration of batch. Packet arrivals is close to the realistic situation in a communication system. Kuda Nageswara Rao et al.(2011) have developed some two node tandem communication network models with batch arrivals at the source and dynamic bandwidth strategy. However the transmission nodes in a communication system are generally in multiple number of series or tandem between the sender and receiver. The assumption of three nodes in series having a predecessor and a successor for the middle node is more generic, appropriate and realistic to the network architectures.

In this paper Steady state analysis of a three stage communication network with dynamic bandwidth allocation and batch arrivals from the source connected to the first node is modeled through embedded Markov chain techniques. Using the difference differential equations the performance measures of the communication network such as the joined probability generating function of the number of packets in each buffer, the probability of emptiness of buffers, mean number of packets in the buffers, mean delays in the buffers, throughput of the nodes are derived explicitly under equilibrium conditions. The performance evaluation of the network model is studies through numerical illustration.

II. Communication Network Model Under Steady State Condition

Steady state analysis of a three stage communication network with dynamic bandwidth allocation and batch arrivals is developed and analyzed. Consider the messages arrive to the first node are converted a random number of packets and stored in the first buffer connected to the first node.

The packets are forwarded to the second buffer connected to the second node after transmitting from the first node. It is further considered that after transmitting from the second node the packets are forwarded to the third buffer connected to the third node. It is assumed that the arrival of packets to the first buffer is in batch with random batch size having the probability mass function $\{\mathbf{Ck}\}$. It is considered that the random transmission is carried with dynamic bandwidth allocation in all the three nodes i.e. the transmission rate at each node is adjusted instantaneously and dynamically depending upon the content of the buffer connected to each node. This can be modeled as the transmission rates are linearly dependent on the content of the buffer. It is assumed that the arrival of packets following compound Poisson process with parameter λ and the number of transmissions at node 1, node 2 and node 3 follow Poisson process with parameters β, δ, θ respectively.

The operating principle of the queue is First in First out (FIFO). The schematic diagram represents the proposed communication network model is shown in Figure 1. For obtaining the performance of a communication network, it is needed to know the function form of the probability mass function of the number of packets that a message can be converted (Ck). Using the difference differential equations, the Joint Probability Generating Function of the number of packets in the first, second and third buffers is derived as

$$
\begin{aligned}
P(Z_1,Z_2,Z_3) = \exp\Bigg\{ & \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^{f} \\
& \left(\frac{\beta}{(\delta-\beta)}\right)^{s-f}(Z_3-1)^{f}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-\delta}\right]^{s-f} \\
& \left[(Z_1-1)+\frac{\beta(Z_2-1)}{\delta-\beta}+\frac{\beta\delta(Z_3-1)}{(\delta-\beta)(\theta-\beta)}\right]^{r-s} \\
& \frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\Bigg\} \text{ for } |Z_1|<1, |Z_2|<1, |Z_3|<1
\end{aligned} \tag{1}
$$

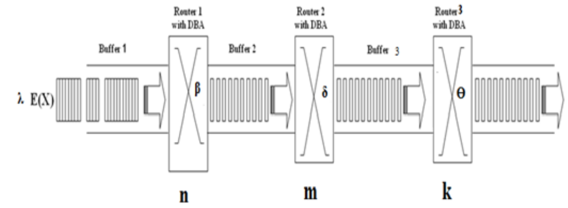

Fig.1 Three Node Tandem Communication network with dynamic bandwidth allocation and batch arrivals

III. Performance Measures Of The Proposedcommunication Network

The probability of emptiness of the whole network is

$$
\begin{aligned}
P_{(0,0,0)} = \exp\Bigg\{ & \lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^{f}\left(\frac{\beta}{\delta-\beta}\right)^{s-f}\left(\frac{-\theta}{\theta-\delta}\right)^{s-f} \\
& \left[-1-\frac{\beta}{\delta-\beta}+\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right]^{r-s}\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\Bigg\}
\end{aligned} \tag{2}
$$

The probability generating function of the number of packets in the first buffer is

$$
P(Z_1) = \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x\binom{x}{r}(Z_1-1)^{r}\left(\frac{1}{\beta r}\right)\right\} \text{ for } |Z_1|<1 \tag{3}
$$

The probability that the first buffer is empty,

$$
P_{0..} = \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x\binom{x}{r}(Z_1-1)^{r}\left(\frac{1}{\beta r}\right)\right\} \tag{4}
$$

The probability generating function of the number of packets in the second buffer is

$$
\begin{aligned}
P(Z_2) = \exp\Bigg\{ & \lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}(Z_2-1)^{r} \\
& \left(\frac{1}{\delta s+\beta(r-s)}\right)\Bigg\}; \text{for } |Z_3|<1
\end{aligned} \tag{5}
$$

The probability that the second buffer is empty

$$
\begin{aligned}
P_{.0.} = \exp\Bigg\{ & \lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r} \\
& \left(\frac{1}{\delta s+\beta(r-s)}\right)\Bigg\}
\end{aligned} \tag{6}
$$

The probability generating function of the number of packets in the third buffer is

$$
\begin{aligned}
P(Z_3) = \exp\Bigg\{ & \lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} c_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \\
& \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^{r}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\Bigg\} \quad ;\text{for } Z_3<1
\end{aligned} \tag{7}
$$

The probability that the third buffer is empty,

$$P_{..0} = \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} c_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^r\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \tag{8}$$

The mean number of packets in the first buffer is

$$L_1 = \frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty} x.C_x\right) \tag{9}$$

The mean number of packets in the second buffer is,

$$L_2 = \frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty} x.C_x\right) \tag{10}$$

The mean number of packets in the third buffer is

$$L_3 = \lambda\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\beta} - \frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\delta} + \frac{\beta\delta}{(\theta-\delta)(\theta-\beta)\theta}\right] \tag{11}$$

The probability that there is at least one packet in the first node is

$$U_1 = 1 - P_{0..} = 1 - \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x \binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\} \tag{12}$$

The probability that there is at least one packet in the second node is

$$U_2 = 1 - P_{.0.} = 1 - \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\} \tag{13}$$

The probability that there is at least one packet in the third node is,

$$U_3 = 1 - P_{..0} = 1 - \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \tag{14}$$

The mean number of packets in the whole network is

$$L = L_1 + L_2 + L_3 \tag{15}$$

Throughput of the first node is

$$Thp1 = \beta U_1 = \beta\left[1 - P_{0..}\right] = \beta\left\{1 - \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\}\right\} \tag{16}$$

Throughput of the second node is

$$Thp2 = \delta U_2 = \delta\left(1 - P_{.0.}\right) = \delta\left(1 - \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\}\right) \tag{17}$$

Throughput of the third node is

$$Thp3 = \theta U_3 = \theta\left(1 - P_{..0}\right) = \theta\left\{1 - \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right\} \tag{18}$$

The mean delay in the first buffer is

$$W_1 = \frac{L_1}{\beta\left(1 - P_{0..}\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty} xC_x\right)}{\beta\left[1 - \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} c_x\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\}\right]} \tag{19}$$

The mean delay in the second buffer is

$$W_2 = \frac{L_2}{\delta\left(1 - P_{.0.}\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty} x.C_x\right)}{\delta\left(1 - \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\}\right)} \tag{20}$$

The mean delay in the third buffer is

$$W_3 = \frac{L_3}{\theta\left(1 - P_{..0}\right)} = \frac{\lambda\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\beta} - \frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\delta} + \frac{\beta\delta}{(\theta-\delta)(\theta-\beta)\theta}\right]}{\theta\left(1 - \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right)} \tag{21}$$

The variance of the number of packets in the first buffer is

$$V_1 = \frac{\lambda}{2\beta}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right) + \frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty} xC_x\right) \tag{22}$$

The variance of the number of packets in the second buffer is

$$V_2 = \lambda\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left(\frac{\beta}{\beta-\delta}\right)^2\left[\left(\frac{1}{2\beta}\right) - 2\left(\frac{1}{\beta+\delta}\right) + \left(\frac{1}{2\delta}\right)\right] + \lambda\left(\sum_{x=1}^{\infty} xC_x\right)1/\delta \tag{23}$$

The variance of the number of packets in the third buffer is

$$\begin{aligned} V_3 = {} & \lambda(\beta\delta)^2\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left\{\left(\frac{1}{(\delta-\beta)(\theta-\beta)}\right)^2\left(\frac{1}{2\beta}\right) - 2\left(\frac{1}{\delta-\beta}\right)^2\right. \\ & \left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]\left(\frac{1}{\beta+\delta}\right) + 2\left(\frac{1}{\theta-\beta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right] \\ & \left(\frac{1}{\beta+\theta}\right) + \left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]^2\left(\frac{1}{2\delta}\right) - 2\left(\frac{1}{\theta-\delta}\right)^2 \\ & \left[\frac{1}{(\delta-\beta)(\theta-\beta)}\right]\left(\frac{1}{\theta+\delta}\right) + \left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]^2\left(\frac{1}{\theta}\right) \\ & \lambda\beta\delta\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{1}{(\delta-\beta)(\theta-\beta)\beta}\right) - \left(\frac{1}{(\delta-\beta)(\theta-\delta)\delta}\right) + \left(\frac{1}{(\theta-\delta)(\theta-\beta)\theta}\right)\right] \end{aligned} \tag{24}$$

The coefficient of variation of the number of packets in the first node is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \tag{25}$$

The coefficient of variation of the number of packets in the second node is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \tag{26}$$

The coefficient of variation of the number of packets in the third node is

$$CV_3 = \frac{\sqrt{V_3}}{L_3} \tag{27}$$

IV. Performance Measures Of The Proposed Network Model With Uniform Batch Size Distribution

It is assumed the batch size of the packets follows a uniform distribution and the probability distribution of the batch size of the packets in a message is $C_k = 1 / \{(b-a) +1\}$, for k= a, a+1, … , b. and the mean number of packets in a message is $\left(\frac{a+b}{2}\right)$ and its variance is $\frac{1}{12}\left[(b-a+1)^2-1\right]$ Substituting the value of C_x in the equation (1), we get the Joint Probability Generating Function of the number of packets in the whole networks is

$$P(Z_1,Z_2,Z_3)=\exp\left\{\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta}{(\delta-\beta)}\right)^{s-f}(Z_3-1)^f\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-\delta}\right]^{s-f}\left[(Z_1-1)+\frac{\beta(Z_2-1)}{\delta-\beta}+\frac{\beta\delta(Z_3-1)}{(\delta-\beta)(\theta-\beta)}\right]^{r-s}\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right\}\text{ for }|Z_1|<1,|Z_2|<1,|Z_3|<1 \quad (28)$$

The probability of emptiness of the whole network is

$$P_{(0,0,0)}=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^{f}\left(\frac{\beta}{\delta-\beta}\right)^{s-f}\left(\frac{-\theta}{\theta-\delta}\right)^{s-f}\left[-1-\frac{\beta}{\delta-\beta}+\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right]^{r-s}\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right\} \quad (29)$$

The probability generating function of the number of packets in the first buffer is

$$P(Z_1)=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\}\text{ for }|Z_1|<1 \quad (30)$$

The probability that the first buffer is empty,

$$P_{0..}=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\} \quad (31)$$

The probability generating function of the number of packets in the second buffer is

$$P(Z_2)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}(Z_2-1)^r\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\};\text{for }|Z_3|<1 \quad (32)$$

The probability that the second buffer is empty

$$P_{.0.}=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\} \quad (33)$$

The probability generating function of the number of packets in the third buffer is

$$P(Z_3)=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^r\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\};\text{for }Z_3<1 \quad (34)$$

The probability that the third buffer is empty,

$$P_{..0}=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^r\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \quad (35)$$

The mean number of packets in the first buffer is

$$L_1=\frac{\lambda}{\beta}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right) \quad (36)$$

The mean number of packets in the second buffer is,

$$L_2=\frac{\lambda}{\beta}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right) \quad (37)$$

The mean number of packets in the third buffer is

$$L_3=\lambda\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left[\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\beta}-\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\delta}+\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)\theta}\right] \quad (38)$$

The probability that there is at least one packet in the first node is

$$U_1=1-P_{0..}=1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\} \quad (39)$$

The probability that there is at least one packet in the second node is

$$U_2=1-P_{.0.}=1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\} \quad (40)$$

The probability that there is at least one packet in the third node is,

$$U_3=1-P_{..0}=1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \quad (41)$$

The mean number of packets in the whole network is
$L=L_1+L_2+L_3$ (42)

Throughput of the first node is

$$Thp1=\beta U_1=\beta\left[1-P_{0..}\right]=\beta\left\{1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1}{\beta r}\right)\right\}\right\} \quad (43)$$

Throughput of the second node is

$$Thp2=\delta U_2=\delta\left(1-P_{.0.}\right)=\delta\left(1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1}{\delta s+\beta(r-s)}\right)\right\}\right) \quad (44)$$

Throughput of the third node is

$$Thp3=\theta U_3=\theta\left(1-P_{..0}\right)=\theta\left\{1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right\} \quad (45)$$

The mean delay in the first buffer is

$$W_1 = \frac{L_1}{\beta\left(1 - P_{0..}\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)}{\beta\left[1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\left(Z_1 - 1\right)^r\left(\frac{1}{\beta r}\right)\right\}\right]} \tag{46}$$

The mean delay in the second buffer is

$$W_2 = \frac{L_2}{\delta\left(1 - P_{.0.}\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)}{\delta\left(1 - \exp.\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1}{\delta s + \beta(r-s)}\right)\right\}\right)} \tag{47}$$

The mean delay in the third buffer is

$$W_3 = \frac{L_3}{\theta\left(1 - P_{..0}\right)} = \frac{\lambda\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\beta} - \frac{\beta\delta}{(\delta-\beta)(\theta-\beta)\delta} + \frac{\beta\delta}{(\theta-\delta)(\theta-\beta)\theta}\right]}{\theta\left(1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1}{\theta f + \delta(s-f) + \beta(r-s)}\right)\right\}\right)} \tag{48}$$

The variance of the number of packets in the first buffer is

$$V_1 = \frac{\lambda}{2\beta}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right) + \frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right) \tag{49}$$

The variance of the number of packets in the second buffer is

$$V_2 = \lambda\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left(\frac{\beta}{\beta-\delta}\right)^2\left[\left(\frac{1}{2\beta}\right) - 2\left(\frac{1}{\beta+\delta}\right) + \left(\frac{1}{2\delta}\right)\right] + \lambda\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(\frac{1}{\delta}\right) \tag{50}$$

The variance of the number of packets in the third buffer is

$$V_3 = \lambda(\beta\delta)^2\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left\{\left(\frac{1}{(\delta-\beta)(\theta-\beta)}\right)^2\left(\frac{1}{2\beta}\right) - 2\left(\frac{1}{\delta-\beta}\right)^2\left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]\left(\frac{1}{\beta+\delta}\right) + 2\left(\frac{1}{\theta-\beta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]\left(\frac{1}{\beta+\theta}\right) + \left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]^2\left(\frac{1}{2\delta}\right) - 2\left(\frac{1}{\theta-\delta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\beta)}\right]\left(\frac{1}{\theta+\delta}\right) + \left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]^2\left(\frac{1}{\theta}\right)\right. \tag{51}$$

$$\lambda\beta\delta\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1}{(\delta-\beta)(\theta-\beta)\beta}\right) - \left(\frac{1}{(\delta-\beta)(\theta-\delta)\delta}\right) + \left(\frac{1}{(\theta-\delta)(\theta-\beta)\theta}\right)\right]$$

V. Performance Analysis Of The Proposed Communication Network

The performance of the proposed network is analyzed through numerical illustration. A set of values of the input parameters are considered for allocation of bandwidth and arrival of packets. After interacting with the internet service provider, it is considered that the message arrival rate (λ) varies from $1x10^4$ messages/sec to $5x10^4$ messages/sec, the number of packets that can be converted from a message varies from 1 to 10. The message arrivals to the buffer are in batches of random size. The batch size is assumed to follow uniform distribution parameters (a, b). The transmission rate of node 1(β) varies from $1x10^4$ packets/sec to 4x104 packets/sec. The packets leave the second node with a transmission rate (δ) which varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec. The packets leave the third node with a transmission rate (θ) which varies from $11x10^4$packets/sec to$14x10^4$ packets/sec. In all the three nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

The probability of network emptiness and different buffers emptiness are computed for different values of a, b λ, β, δ, θ. It is observed that the probability of emptiness of the communication network and the three buffers are highly sensitive. When the batch distribution parameter (a) varies from$1x10^4$ packets/sec to $5x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.016 to 0.004 when other parameters are fixed at (6, 1, 1.1, 2, 1) for (a, b, λ, β, δ, θ). The same phenomenon is observed with respect to the first and second nodes. The probability of emptiness of the first, second and third buffers decrease from 0.221 to 0.083 and 0.936 to 0.883 and 0.492 to 0.221 respectively.

When the batch size distribution parameter (b) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.121 to 0.068 when other parameters are fixed at (5, 1, 1.1, 2, 1) for (a, λ, β, δ, θ). The same phenomenon is observed with respect to the first, second and third node. The probability of emptiness of the first, second and third buffers decrease from 0.492 to 0.105, 0.811 to 0.791 and 0.718 to 0.284 respectively. The influence of arrival of messages on system emptiness is also studied. As the arrival rate (λ) varies from $0.5x10^4$ messages/sec to $2.0x10^4$ messages/sec, the probability of emptiness of the network decreases from 0.510 to 0.471 when other parameters are fixed at (5, 6, 1.1, 2, 1) for (a, b, β, δ, θ). The same phenomenon is observed with respect to the first, second and third nodes. The probability of emptiness of the first, the second and third buffer decrease from 0.025 to 0.012, 0.988 to 0.979 and 0.064 to 0.058 respectively. When the transmission rate (β) of node1 varies from $1.2x10^4$ packets/sec to $1.8x10^4$ packets/sec, the probability of emptiness of the network decrease from 0.740 to 0.442, first, second buffers are constant and third buffer is decreases from 0.093 to 0.043 when other parameters remain fixed at (5, 6, 1, 2, 1) for (a, b, λ, δ, θ).

When the transmission rate of node 2 **(**δ) varies from $2.2x10^4$ packets/sec to $2.8x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.916 to 0.159 the second buffer decreases from 0.064 to 0.049 and third buffer remains constant, when other parameters fixed at (5, 6, 1, 1.1, 1) for (a, b, λ, β, θ). When the transmission rate of node 3 (δ) varies from $1.2x10^4$ packets/sec to $1.8x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.324 to 0.213, first node is constant, second buffer decreases from 0.901 to 0.861 and third buffer is constant, when other parameters remain fixed at (5, 6, 1, 1.1, 2) for (a, b, λ, β, θ). The mean number of packets and the utilization of the network are computed for different values of a, b, λ, β, δ, θ. Values of probability of emptiness, mean number packets, throughputs and mean delays are in the three buffers are given in Table.1 and the relationship between batch size distribution parameter a, b Vs Mean Number of Packets, Throughputs, Mean delays and Emptiness in the buffers at nodes 1, 2 and 3 are shown in Figure 2.

Table 1 Values of probability of emptiness mean number packets, throughputs and mean delays are in the three buffers

a	b	$\lambda^{\#}$	$\beta^{\$}$	$\delta^{\$}$	$\Theta^{\$}$	$P_{000}(t)$	$P_{0..}(t)$	$P_{.0.}(t)$	$P_{..0}(t)$	L_1	L_2	L_3	Ln	Thp1	Thp2	Thp3	W_1	W_2	W_3
1	6	1	1.1	2	1	0.016	0.221	0.936	0.492	0.20	0.20	0.560	0.443	0.856	0.128	0.508	0.233	1.562	1.102
2	6	1	1.1	2	1	0.014	0.174	0.923	0.449	0.20	0.20	0.567	0.665	0.908	0.154	0.551	0.220	1.298	1.029
3	6	1	1.1	2	1	0.011	0.127	0.905	0.350	0.20	0.20	0.578	0.887	0.960	0.19	0.65	0.208	1.052	0.889
4	6	1	1.1	2	1	0.004	0.083	0.883	0.221	0.20	0.20	0.597	1.108	1.008	0.234	0.779	0.198	0.854	0.766
5	**1**	1	1.1	2	1	0.121	0.492	0.819	0.718	0.40	0.43	1.417	2.217	0.558	0.362	0.282	0.715	1.187	5.024
5	**2**	1	1.1	2	1	0.105	0.221	0.811	0.649	0.81	0.62	1.608	2.308	0.856	0.378	0.351	0.945	1.640	4.581
5	**3**	1	1.1	2	1	0.098	0.143	0.803	0.396	0.933	0.925	1.801	2.439	0.942	0.394	0.604	0.989	2.347	2.981
5	**4**	1	1.1	2	1	0.068	0.105	0.791	0.284	0.982	0.992	1.992	2.554	0.984	0.418	0.716	0.997	2.373	2.782
5	6	**0.5**	1.1	2	1	0.510	0.025	0.988	0.064	0.025	0.25	0.089	0.364	1.072	0.024	0.936	0.023	10.416	0.095
5	6	**1**	1.1	2	1	0.497	0.021	0.986	0.063	0.026	0.26	0.090	0.378	1.076	0.028	0.937	0.024	9.285	0.096
5	6	**1.5**	1.1	2	1	0.484	0.017	0.983	0.061	0.027	0.28	0.096	0.393	1.081	0.034	0.939	0.024	8.235	0.102
5	6	**2.0**	1.1	2	1	0.471	0.012	0.979	0.058	0.029	0.30	0.099	0.407	1.086	0.042	0.942	0.026	7.142	0.105
5	6	1	**1.2**	2	1	0.740	0.049	0.933	0.110	0.048	0.087	0.691	0626	1.141	0.134	0.89	0.042	0.649	0.776
5	6	1	**1.4**	2	1	0.642	0.049	0.933	0.093	0.048	0.091	0.766	0.901	1.331	0.134	0.907	0.036	0.679	0.844
5	6	1	**1.6**	2	1	0.581	0.049	0.933	0.062	0.048	0.095	0.892	1.005	1.521	0.134	0.938	0.031	0.708	0.950
5	6	1	**1.8**	2	1	0.442	0.049	0.933	0.036	0.048	0.099	0.998	1.219	1.711	0.134	0.964	0.028	0.738	1.035
5	6	1	1.1	**2.2**	1	0.916	0.064	0.988	0.043	0.063	0.625	0.952	0.693	1.029	0.026	0.957	0.061	23.674	0.994
5	6	1	1.1	**2.4**	1	0.732	0.058	0.988	0.041	0.068	0.668	0.958	0.696	1.036	0.028	0.959	0.065	23.194	0.998
5	6	1	1.1	**2.6**	1	0.411	0.053	0.988	0.038	0.072	0.691	0.963	0.701	1.041	0.031	0.962	0.069	22.147	1.001
5	6	1	1.1	**2.8**	1	0.159	0.049	0.988	0.032	0.078	0.718	0.981	0.754	1.046	0.033	0.968	0.074	21.369	1.013
5	6	1	1.1	2	**1.2**	0.324	0.049	0.901	0.043	0.048	0.058	0.493	0.599	1.046	0.198	1.148	0.045	0.292	0.429
5	6	1	1.1	2	**1.4**	0.301	0.049	0.895	0.043	0.051	0.067	0.549	0.665	1.046	0.21	1.339	0.048	0.319	0.409
5	6	1	1.1	2	**1.6**	0.245	0.049	0.886	0.043	0.055	0.077	0.622	0.747	1.046	0.228	1.531	0.052	0.337	0.406
5	6	1	1.1	2	**1.8**	0.213	0.049	0.861	0.043	0.068	0.087	0.719	0.853	1.046	0.278	1.722	0.065	0.352	0.401

 # =Multiples of 10,000 Messages/sec, $= Multiples of 10,000 Packets/sec

As the batch size distribution parameter (a) varies from 1 to 4, the first buffer, second buffer and third buffer the network average content increase from 0.443 $x10^4$ packets to $1.108x10^4$ packets, first, second buffer mean number of packets are constant and the third buffer mean number of packets increases from $0.560x10^4$ packets to $0.597x10^4$ packets when other parameters remain fixed. As the batch size distribution parameter (b) varies from 1 to 4, the first buffer, second buffer, third buffer and the network average content increases from 0.40 $x10^4$ packets to 0.98 $x10^4$ packets, from $0.43x10^4$ packets to 0.992 $x10^4$ packets, from 1.417 $x10^4$ packets to 1.992 $x10^4$ packets and from 2.217 $x10^4$ packets to 2.554 $x10^4$ packets respectively when other parameters remain fixed.

As the arrival rate of messages (λ) varies from 0.5 $x10^4$ messages/sec to $2.0x10^4$ messages/sec, the mean number of packets in the first buffer, second buffer, third buffer and in the network increase from 0.025 $x10^4$ packets to 0.029 $x10^4$ packets, from 0.25 $x10^4$ packets to 0.30 $x10^4$ packets, from $0.089vx10^4$ packets to 0.099 $x10^4$ packets, from 0.346 $x10^4$ packets to 0.407 $x10^4$ packets respectively when other parameters remain fixed at (5, 6, 1.1, 2, 1) for (a, b, β, δ, θ). As the transmission rate of node 1 (β) varies from $1.2x10^4$ messages/sec to $1.8x10^4$ packets/sec, the first buffer is constant, second and third buffers are increases from $0.087x10^4$ packets to 0.099 $x10^4$ packets and 0.691 $x10^4$ to 0.998 $x10^4$ respectively when other parameters remain fixed.

As the transmission rate of node 2 (δ) varies from $2.2x10^4$ packets/sec to $2.8x10^4$ packets/sec, first buffer increases from 0.063 $x10^4$ packets to 0.078 $x10^4$ packets, the second, third buffers and the network average content increases from 0.625 $x10^4$ packets to 0.718 $x10^4$ packets, from 0.952 $x10^4$ packets to 0.981 $x10^4$ packets and 0.693 $x10^4$ packets to 0.754 $x10^4$ packets respectively when other parameters remain fixed. As the transmission rate of node 3 (θ) varies from $1.2x10^4$ packets/sec to $1.8x10^4$ packets/sec, the first, second, third buffers and the network average content increase from 0.48 $x10^4$ packets to 0.068 $x10^4$ packets , from 0.058 $x10^4$ packets to 0.087 $x10^4$ packets, form 0.493 $x10^4$ packets to 0.719 $x10^4$ packets and from 0.599 $x10^4$ packets to 0.853 $x10^4$ packets respectively when other parameters remain fixed.

It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here the arrival rate of messages (λ) increase, the utilization of all the three nodes increase for fixed values of the other parameters. As the batch size distribution parameters (a) and (b) increase, the utilization of all the three nodes increase when the other parameters are fixed. It is also noticed that as the transmission rate of node 1(β), node 2 (δ) are constant and the third node increases. Therefore in the communication network, dynamic bandwidth allocation strategy is necessary for control of congestion, efficient utilization of different nodes and to maintain satisfactory quality of service (QoS) with optimum speed.

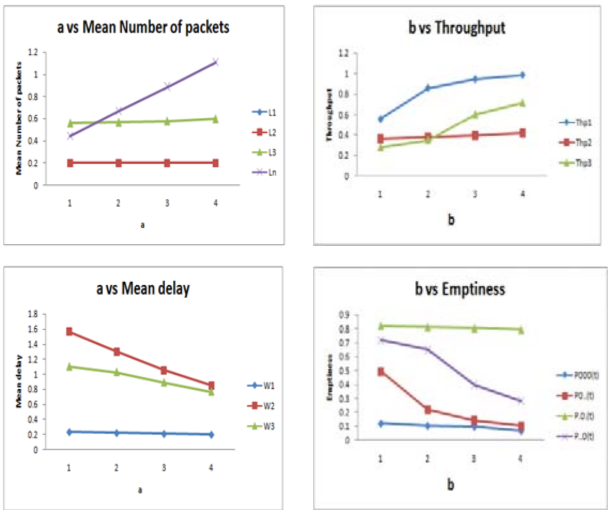

Fig. 2 Batch size distribution parameter a, b Vs Mean Number of Packets, Throughputs, Mean delays and Emptiness in the buffers at nodes 1, 2 and 3

The throughput and the average delay of the network are computed for different values of a, b, λ, β, δ, θ and the values of mean delays are given in Table 1. As the batch size distribution parameter (a) varies from 1 to 4 the throughput of the first, second and third nodes increases from 0.856 $x10^4$ packets to 1.008 $x10^4$ packets, 0.128 $x10^4$ packets to 0.234 $x10^4$ packets and 0.508 $x10^4$ packets to 0.779 $x10^4$ packets respectively when other parameters remain fixed at (6, 1, 1.1, 2, 1) for (b, λ, β, δ, θ). As the batch size distribution parameter (b) varies from 1 to 4, the throughput of the first, second and third nodes increases from

0.558 $x10^4$ packets to 0.984 $x10^4$ packets, 0.362 $x10^4$ packets to 0.418 $x10^4$ packets and 0.0282 $x10^4$ packets to 0.716 $x10^4$ packets respectively when other parameters remain fixed at (5, 1, 1.1, 2, 1) for (a, λ, β, δ, θ). As the arrival rate (λ) varies from 0.5 to 2.0 the throughput of the first, second and third nodes increase from 1.072$x10^4$ packets to 1.086 $x10^4$ packets, from 0.024 $x10^4$ packets to 0.042 $x10^4$ packets and 0.936 $x10^4$ packets to 0.942 $x10^4$ packets respectively when other parameters remain fixed at (5, 6, 1.1, 2, 1) for (a, b, β, δ, θ).

As the transmission rate (β) of node1 varies from 1.2$x10^4$ packets/sec to 1.8$x10^4$ packets/sec, the throughput of first node increases from 1.141 $x10^4$ packets to 1.711$x10^4$ packets, second node constant and the third node increases from 0.89$x10^4$ packets to 0.964 $x10^4$ packets, when other parameters remain fixed at (5, 6, 1, 2, 1) for (a, b, λ, δ, θ). As the transmission rate of node 2(β) varies from 2.2$x10^4$ packets/sec to 2.8$x10^4$ packets/sec, the throughput of first, second and third nodes increase from 1.029$x10^4$ packets to 1.041 $x10^4$ packets, from 0.026$x10^4$ packets to 0.033 $x10^4$ packets and from 0.957$x10^4$ packets to 0.968 $x10^4$ packets respectively when other parameters remain fixed at (5, 6, 1, 1.1, 1) for (a, b, λ, β, θ). As the transmission rate of node 3(θ) varies from 1.2$x10^4$ packets/sec to 1.8$x10^4$ packets/sec, the throughput of first node remains constant, second node increase from 0.198$x10^4$ packets to 0.278 $x10^4$ packets and third node increase from 1.148$x10^4$packets to 1.722 $x10^4$ packets respectively when other parameters remain fixed at (5, 6, 1, 1.1, 2) for (a, b, λ, β, δ).

As the batch size distribution parameter (a) varies from 1 to 4, the mean delay of the first buffer decrease from 0.233μs to 0.198μs ,the second and third buffers are decreases from 1.562μs to 0.854μs and from 1.102μs to 0,766μs respectively, when other parameters remain fixed (6, 1, 1.1, 2, 1) for (b, λ, β, δ, θ). As the batch size distribution parameter (b) varies from 1 to 4, the mean delay of the first increases from, 0.715μs to 0.997μs ,second increase from 1.187μs to 2.373μs and third buffer decreases from 5.024μs to 2.782μs respectively, when other parameters remain fixed (5, 1, 1.1, 2, 1) for (a, λ, β, δ, θ). When the arrival rate (λ) varies from 0.5$x10^4$ messages/sec to 2.0$x10^4$ messages/sec, the mean delay of the first buffer increase from 0.023μs to 0.026μs , second buffer decrease from 10.416μs to 7.142μs and third buffer increases from 0.096μs to 0.105μs respectively, when other parameters remain fixed (5, 6, 1.1, 2, 1) for (a, b, β, δ, θ). As the transmission rate of node 1 (β) varies from 1.2$x10^4$messages/sec to 1.8$x10^4$ messages/sec, the mean delay of the first node decreases from 0.042μs to 0.028 μs , second and third buffer increases from 0.649μs to 0.738μs, and from 0.776μs to 1.035μs when other parameters remain fixed at (5, 6, 1, 2, 1) for (a, b, λ, δ, θ). As the transmission rate of node 2 (δ) varies from 2.2$x10^4$ packets/sec to 2.8$x10^4$ packets/sec, the mean delay of the first buffer increases from 0.061 μs to 0.074 μs, second buffer decreases from 23.674μs to 21.369μs and the third buffer increases from 0.994μs to 1.001 μs when other parameters remain fixed at (5, 6, 1, 1.1, 1) for (a, b, λ, β, θ). As the transmission rate of node 3 (θ) varies from 1.2$x10^4$ packets/sec to 1.8$x10^4$ packets/sec, the mean delay of the first buffer increases from 0.045μs to 0.065μs, second buffer increases from 0.292μs to 0.352μs, and third buffer decreases from 0.429μs to 0.401μs when other parameters remain fixed at (5, 6, 1, 1.1, 2) for (a, b, λ, β, δ).

VI. CONCLUSIONS

In this paper, Steady state analysis of a three stage communication network with dynamic bandwidth allocation and batch arrivals is developed and analyzed. Here, the dynamic bandwidth allocation strategy insists for the instantaneous change in rate of transmission of the nodes depending upon the content of the buffers connected to them. The emphasis of this communication network is on the batch arrivals of packets to the initial node with random size. The performance of the statistical multiplexing is measured by approximating the arrival process with a compound Poisson process and the transmission process. This is chosen such that the statistical characteristics of the communication network identically matches with Poisson process and uniform distribution. A communication network model with batch arrivals is more close to the practical transmission behavior in most of the communication systems. It is observed that the dynamic bandwidth allocation strategy and the parameters of batch size distribution have a significant impact on the performance measures of the network. It is further observed that steady state analysis of the Communication network will approximate the performance measures more close to the practical situation. It improves the Quality of Service (QoS) by effective utilization of the bandwidth and avoids the congestion in the network.

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